

Various Functions Questions

1. Let the function $g(a, b) = f(a) + f(b)$.

For which function f below will $g(a + b, a + b) = g(a, a) + g(b, b)$?

- A: $x + 3$
- B: x^2
- C: $|x|$
- D: $1/x$
- E: $x/4$

2. The function f is defined for all positive integers a and b by the following rule:
 $f(a, b) = (a + b) / \text{GCF}(a, b)$, where $\text{GCF}(a, b)$ is the greatest common factor of a and b . If $f(10, x) = 11$, what is the value of x ?

- (1) x is a square of an integer
- (2) The sum of the distinct prime factors of x is a prime number

3. A sequence is given by the rule $a_n = |a_{(n-2)}| - |a_{(n-1)}|$ for all $n \geq 3$,
 where $a_1 = 0$ and $a_2 = 3$.

A function S_n is defined as the sum of all the terms of the sequence from its beginning through a_n . For instance, $S_4 = a_1 + a_2 + a_3 + a_4$. What is S_{101} ?

- (A) -3
- (B) 0
- (C) 3
- (D) 201
- (E) 303

4. For which of the following functions does $f(x) = f(2 - x)$?

- A. $f(x) = x + 2$
- B. $f(x) = 2x - x^2$
- C. $f(x) = 2 - x$
- D. $f(x) = (2 - x)^2$
- E. $f(x) = x^2$

5. For every positive even integer n , the function $h(n)$ is defined to be the product of all the even integers from 2 to n , inclusive. If p is the smallest prime factor of $h(100) + 2$, then p is?

A. between 2 and 20
B. between 10 and 20
C. between 20 and 30
D. between 30 and 40
E. 2

6. For which of the following functions is $f(a+b)=f(b)+f(a)$ for all positive numbers a and b ?

A. $f(x)=x^2$
B. $f(x)=x+1$
C. $f(x)=\sqrt{x}$
D. $f(x)=2/x$
E. $f(x)=-3x$

7. For which of the following functions f is $f(x) = f(1-x)$ for all x ?

A. $f(x) = 1 - x$
B. $f(x) = 1 - x^2$
C. $f(x) = x^2 - (1 - x)^2$
D. $f(x) = x^2 (1 - x)^2$
E. $f(x) = x/(1 - x)$

8. The function f is defined for each positive three-digit integer n by $f(n) = 2^x 3^y 5^z$, where x , y and z are the hundreds, tens, and units digits of n , respectively. If m and v are three-digit positive integers such that $f(m)=9f(v)$, then $m-v=?$

(A) 8
(B) 9
(C) 18
(D) 20
(E) 80

9. If $f(x) = \frac{125}{x^3}$, what is the value of $f(5x) * f(\frac{x}{5})$ in terms of $f(x)$?

A. $(f(x))^2$

B. $f(x^2)$

C. $(f(x))^3$

D. $f(x^3)$

E. $f(125x)$

10. The function f is defined by $f(x) = -1/x$ for all nonzero numbers x . If $f(a) = -1/2$ and $f(ab) = 1/6$, then $b =$
- A. 3
 - B. $1/3$
 - C. $-1/3$
 - D. -3
 - E. -12
11. The function $f(m)$ is defined for all positive integers m as the product of $m + 4$, $m + 5$, and $m + 6$. If n is a positive integer, then $f(n)$ must be divisible by which one of the following numbers?
- (A) 4
 - (B) 5
 - (C) 6
 - (D) 7
 - (E) 11
12. The function $p(n)$ on non-negative integer n is defined in the following way: the units digit of n is the exponent of 2 in the prime factorization of $p(n)$, the tens digit is the exponent of 3, and in general, for positive integer k , the digit in the $10^{(k-1)}$ th place of n is the exponent on the k th smallest prime (compared to the set of all primes) in the prime factorization of $p(n)$. For instance, $p(102) = 20$, since $20 = (5^1)(3^0)(2^2)$. What is the smallest positive integer that is not equal to $p(n)$ for any permissible n ?
- (A) 1
 - (B) 29
 - (C) 31
 - (D) 1,024
 - (E) 2,310
13. The function f is defined for all positive integers n by the following rule. $f(n)$ is the number of positive integers each of which is less than n and has no positive factor in common with n other than 1. If p is any prime, number then $f(p) =$
- A. $p-1$
 - B. $p-2$
 - C. $(p+1)/2$
 - D. $(p-1)/2$
 - E. 2

14. The function $F(n)$ is defined as the product of all the consecutive positive integers between 1 and n^2 , inclusive, whereas the function $G(n)$ is defined as the product of the squares of all the consecutive positive integers between 1 and n , inclusive. The exponent on 2 in the prime factorization of $F(3)/G(3)$ is
- (A) 1
 - (B) 2
 - (C) 3
 - (D) 4
 - (E) 5
15. For every even positive integer m , $f(m)$ represents the product of all even integers from 2 to m , inclusive. For example, $f(12) = 2 \times 4 \times 6 \times 8 \times 10 \times 12$. What is the greatest prime factor of $f(24)$?
- (A) 23
 - (B) 19
 - (C) 17
 - (D) 13
 - (E) 11
16. A function $V(a, b)$ is defined for positive integers a, b and satisfies $V(a, a) = a$, $V(a, b) = V(b, a)$, $V(a, a+b) = (1 + a/b) V(a, b)$. The value represented by $V(66, 14)$ is ?
- (A) 364
 - (B) 231
 - (C) 455
 - (D) 472
 - (E) None of the foregoing
17. For all integers n , the function f is defined by $f(n) = a^n$, where a is a constant. What is the value of $f(1)$?
- (1) $f(2) = 100$
 - (2) $f(3) = -1,000$
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